Mappings On Soft Classes

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ABSTRACT. In this paper, we define the notion of a mapping on soft classes and study several properties of images and inverse images of soft sets supported by examples and counterexamples. Finally, these notions have been applied to the problem of medical diagnosis in medical expert systems.

Keywords : Soft set; Soft class; Mapping on soft classes; Image of soft set; Inverse image of a soft set; Medical diagnosis in medical expert systems.

1. Introduction

To solve complicated problems in economics, engineering and environment, we cannot successfully use classical methods because of different kinds of incomplete knowledge, typical for those problems. There are four theories: Theory of Probablity, Fuzzy Set Theory (FST) [18], Interval Mathematics and Rough Set Theory (RST) [13], which we can consider as mathematical tools for dealing with imperfect knowledge. All these tools require the pre-specification of some parameter to start with, e.g. probablity density function in Probablity Theory, membership function in FST and an equivalence relation in RST. Such a requirement, seen in the backdrop of imperfect or incomplete knowledge, raises many problems. At the same time, incomplete knowledge remains the most glaring characteristic of humanistic systems – systems exemplified by economic systems, biological systems, social systems, political systems, information systems and, more generally, man-machine systems of various types.

Noting problems in parameter specification Molodtsov [11] introduced the notion of soft set to deal with problems of incomplete information. Soft Set Theory (SST) does not require the specification of a parameter, instead it accommodates approximate descriptions of an object as its starting point. This makes SST a natural mathematical formalism for approximate reasoning. We can use any parametrization we prefer: with the help of words, sentences, real numbers, functions, mappings, and so on. This means that the problem of setting the membership function or any similar problem does not arise in SST.

SST has seminal links with rough set technique of automated knowledge discovery. Soft set being collection of information granules, bears a close resemblance with rough sets.

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A rough set [13] is defined to be a set given by an upper and a lower approximation sets from a universe of information granules. Aktas and Cagman [1] have shown that, both, an arbitrary rough set or an arbitrary fuzzy set may be expressed as a soft set. Hence Soft Set Theory is more general a set up than RST and/or FST. Links between soft sets and information systems and hence to Rough Set Theory, have been further studied in [14, 16, 19]. On the other hand, techniques from RST have been found applicable to SST, due to the affinity of both approaches. Maji, Biswas and Roy [9] applied the technique of knowledge reduction to the information table induced by a soft set. Another parametrization reduction of soft set was proposed in [2, 3]. Recently Z. Kong *et.al.* has also proposed yet another novel method of parameter reduction in [6].

Applications of Soft Set Theory in other disciplines and real life problems are now catching momentum. Molodtsov [11] successfully applied the Soft Set Theory into several directions, such as smoothness of functions, Riemann-integration, Perron integration, Theory of Probability, Theory of Measurement and so on. Kovkov *et.al.* [7] has found promising results by applying soft sets to Optimization Theory, Game Theory and Operations Research. Maji and Roy [9] applied soft sets in a multicriteria decision making (MCDM) problem. It is based on the notion of knowledge reduction of rough sets. Mushrif and Sengupta [12] based their algorithm for natural texture classification on soft sets. This algorithm has a low computational complexity when compared to a Bayes technique based method for texture classification. Zou and Xia [19] have exploited the link between soft sets and data analysis in incomplete information systems.

In this paper, we first introduce the notion of mapping on soft classes. Soft classes are collections of soft sets (Definition 7). We also define and study the properties of soft images and soft inverse images of soft sets, and support them with examples and counterexamples. Finally, these notions have been applied to the problem of medical diagnosis in medical expert systems.

2. Preliminaries

First we recall basic definitions and results.

Definition 1. *[11] A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.*
In other words, a soft set over X is a parametrized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly a soft set is not a set in ordinary sense.

Definition 2. *[14] For two soft sets (F, A) and (G, B) over X, we say that (F, A) is a soft subset of (G, B), if*
(i) $A \subseteq B$, and
(ii) $\forall\ \varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$.
We write $(F, A) \widetilde{\subseteq} (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \widetilde{\supseteq} (G, B)$.

Definition 3. *[10] Union of two soft sets (F, A) and (G, B) over the common universe X*

is the soft set (H, C)*, where* $C = A \cup B$*, and* $\forall\, \varepsilon \in C$*,*

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B, \\ G(\varepsilon), & if \quad \varepsilon \in B - A, \\ F(\varepsilon) \cup G(\varepsilon), & if \quad \varepsilon \in A \cap B. \end{cases}$$

We write $(F, A)\ \tilde{\cup}\ (G, B) = (H, C)$*.*

Maji, Biswas and Roy defined the intersection of two soft sets as:

Definition 4. *[10] Intersection of two soft sets* (F, A) *and* (G, B) *over* X *is a soft set* (H, C)*, where* $C = A \cap B$*, and* $\forall\, \varepsilon \in C, H(\varepsilon) = F(\varepsilon)$ *or* $G(\varepsilon)$*, (as both are same set), and is written as* $(F, A)\tilde{\cap}(G, B) = (H, C)$*.*

Pei and Miao pointed out that generally $F(\varepsilon)$ and $G(\varepsilon)$ may not be identical and thus revised the above definition as:

Definition 5. *[14] Let* (F, A) *and* (G, B) *be two soft sets over* X*. Intersection (also called bi-intersction by Feng et.al. [4]) of two soft sets* (F, A) *and* (G, B) *is a soft set* (H, C)*, where* $C = A \cap B$*, and* $\forall\, \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$*. We write* $(F, A)\tilde{\cap}(G, B) = (H, C)$*.*

We further point out that in Definition 5, $A \cap B$ must be nonempty to avoid the degenerate case. Hence the definition 5 is improved as:

Definition 6. *Let* (F, A) *and* (G, B) *be two soft sets over* X *with* $A \cap B \neq \phi$*. Intersection of two soft sets* (F, A) *and* (G, B) *is a soft set* (H, C)*, where* $C = A \cap B$*, and* $\forall\, \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$*. We write* $(F, A)\tilde{\cap}(G, B) = (H, C)$*.*

3. Mappings on Soft Classes

First we define:

Definition 7. *Let* X *be a universe and* E *a set of attributes. Then the collection of all soft sets over* X *with attributes from* E *is called a soft class and is denoted as* (X, E)*.*

Definition 8. *Let* (X, E) *and* (Y, E') *be soft classes. Let* $u : X \to Y$ *and* $p : E \to E'$ *be mappings. Then a mapping* $f : (X, E) \to (Y, E')$ *is defined as: for a soft set* (F, A) *in* (X, E)*,* $(f(F, A), B)$*,* $B = p(A) \subseteq E'$ *is a soft set in* (Y, E') *given by*

$$f(F, A)(\beta) = \begin{cases} u\left(\displaystyle\bigcup_{\alpha \in p^{-1}(\beta) \cap A} F(\alpha)\right), & \text{if } p^{-1}(\beta) \cap A \neq \phi, \\ \phi, & \text{otherwise}, \end{cases}$$

for $\beta \in B \subseteq E'$*.* $(f(F, A), B)$ *is called a soft image of a soft set* (F, A)*. If* $B = E'$*, then we shall write* $f((F, A), E')$ *as* $f(F, A)$*.*

Definition 9. *Let $f:(X,E)\to(Y,E')$ be a mapping from a soft class (X,E) to another soft class (Y,E'), and (G,C), a soft set in soft class (Y,E'), where $C\subseteq E'$. Let $u:X\to Y$ and $p:E\to E'$ be mappings. Then $\left(f^{-1}(G,C),D\right)$, $D=p^{-1}(C)$, is a soft set in the soft class (X,E), defined as:*

$$f^{-1}(G,C)(\alpha)=\begin{cases} u^{-1}(G(p(\alpha))), & p(\alpha)\in C, \\ \phi, & otherwise \end{cases}$$

for $\alpha\in D\subseteq E$. $\left(f^{-1}(G,C),D\right)$ is called a soft inverse image of (G,C). Hereafter we shall write $\left(f^{-1}(G,C),E\right)$ as $f^{-1}(G,C)$.

Example 10. *Let $X=\{a,b,c\}$, $Y=\{x,y,z\}$, $E=\{e_1,e_2,e_3,e_4\}$, $E'=\{e_1',e_2',e_3'\}$ and (X,E), (Y,E'), soft classes. Define $u:X\to Y$ and $p:E\to E'$ as:*

$$\begin{aligned} u(a) &= y,\ u(b)=z,\ u(c)=y, \\ p(e_1) &= e_3',\ p(e_2)=e_3',\ p(e_3)=e_2',\ p(e_4)=e_3'. \end{aligned}$$

Choose two soft sets over X and Y respectively as:

$$\begin{aligned} (F,A) &= \{e_2=\{\},\ e_3=\{a\},\ e_4=\{a,b,c\}\}, \\ (G,C) &= \{e_1'=\{x,z\},\ e_2'=\{y\}\}. \end{aligned}$$

Then the mapping $f:(X,E)\to(Y,E')$ is given as: for a soft set (F,A) in (X,E), $(f(F,A),B)$ where $B=p(A)=\{e_2',e_3'\}$ is a soft set in (Y,E') obtained as follows:

$$\begin{aligned} f(F,A)\,e_2' &= u\left(\bigcup F(\{e_3\})\right)=u(\{a\})=\{y\}, \quad (\text{since } p^{-1}(e_2')\cap A=\{e_3\}), \\ f(F,A)\,e_3' &= u\left(\bigcup_{\alpha\in p^{-1}(e_3')\cap A} F(\alpha)\right)=u(\{F(e_2)\cup F(e_4)\}) \\ &= u(\{\}\cup\{a,b,c\})=u(\{a,b,c\})=\{y,z\},\ (\text{since } p^{-1}(e_3')\cap A=\{e_2,e_4\}). \end{aligned}$$

Hence

$$(f(F,A),B)=\{\ e_2'=\{y\},\ e_3'=\{y,z\}\}.$$

Next for the soft images, we have

$$f^{-1}(G,C)\,e_3=u^{-1}(G(p(e_3)))=u^{-1}(G(e_2'))=u^{-1}(\{y\})=\{a,c\},$$

where $D=p^{-1}(C)=\{e_3\}$.

Hence, we have $\left(f^{-1}(G,C),D\right)=\{e_3=\{a,c\}\}$.

Definition 11. *Let $f:(X,E)\to(Y,E')$ be a mapping and (F,A), (G,B) soft sets in (X,E). Then for $\beta\in E'$, soft union and intersection of soft images of (F, A) and (G, B) in (X, E) are defined as :*

$$\begin{aligned} \left(f(F,A)\ \tilde{\cup}\ f(G,B)\right)\beta &= f(F,A)\,\beta\cup f(G,B)\,\beta, \\ \left(f(F,A)\ \tilde{\cap}\ f(G,B)\right)\beta &= f(F,A)\,\beta\cap f(G,B)\,\beta. \end{aligned}$$

Definition 12. *Let $f:(X,E)\to(Y,E')$ be a mapping and (F,A), (G,B) soft sets in.(Y,E'). Then for $\alpha\in E$, soft union and intersection of soft inverse images of soft sets $(F,A),(G,B)$ are defined as:*

$$\begin{aligned}\left(f^{-1}(F,A)\ \widetilde{\cup}\ f^{-1}(G,B)\right)\alpha &= f^{-1}(F,A)\,\alpha\cup f^{-1}(G,B)\,\alpha,\\ \left(f^{-1}(F,A)\ \widetilde{\cap}\ f^{-1}(G,B)\right)\alpha &= f^{-1}(F,A)\,\alpha\cap f^{-1}(G,B)\,\alpha.\end{aligned}$$

Remark 13. *Note that the null (resp. absolute) soft set as defined by Maji et.al. [10], is not unique in a soft space (X,E), rather it depends upon $A\subseteq E$. Therefore, we denote it by $\widetilde{\Phi}_A$ (resp. $\widetilde{X}_A$). If $A=E$, then we denote it simply by $\widetilde{\Phi}$ (resp. $\widetilde{X}$), which is unique null (resp. absolute) soft set, called full null (resp. full absolute) soft set.*

Theorem 14. *Let $f:(X,E)\to(Y,E')$, $u:X\to Y$ and $p:E\to E'$ be mappings. Then for soft sets (F,A), (G,B) and a family of soft sets (F_i,A_i) in the soft class (X,E), we have:*
(1) $f\left(\widetilde{\Phi}\right)=\widetilde{\Phi}$.
(2) $f\left(\widetilde{X}\right)\ \widetilde{\subseteq}\ \widetilde{Y}$.
(3) $f\left((F,A)\,\widetilde{\cup}\,(G,B)\right)=f(F,A)\,\widetilde{\cup} f(G,B)$.
In general $f\left(\widetilde{\bigcup_i}(F_i,A_i)\right)=\widetilde{\bigcup_i} f(F_i,A_i)$.
(4) $f\left((F,A)\,\widetilde{\cap}\,(G,B)\right)\ \widetilde{\supseteq}\ f(F,A)\,\widetilde{\cap} f(G,B)$,
In general $f\left(\widetilde{\bigcap_i}(F_i,A_i)\right)\widetilde{\subseteq}\widetilde{\bigcap_i} f(F_i,A_i)$.
(5) *If* $(F,A)\ \widetilde{\subseteq}\ (G,B)$, *then* $f(F,A)\ \widetilde{\subseteq}\ f(G,B)$.

Proof. We only prove $(3)-(5)$.
(3) For $\beta\in E'$, we show that $f\left((F,A)\,\widetilde{\cup}\,(G,B)\right)\beta=\left(f(F,A)\ \widetilde{\cup}\ f(G,B)\right)\beta$. Consider

$$\begin{aligned} f\left((F,A)\,\widetilde{\cup}\,(G,B)\right)\beta &= f(H,A\cup B)\,\beta \quad \text{(say)}\\ &= \begin{cases} u\left(\displaystyle\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cup B)} H(\alpha)\right), & \text{if } p^{-1}(\beta)\cap(A\cup B)\neq\phi,\\ \phi, & \text{otherwise,}\end{cases}\\ \text{where } H(\alpha) &= \begin{cases} F(\alpha), & \alpha\in A-B\\ G(\alpha), & \alpha\in B-A\\ F(\alpha)\cup G(\alpha), & \alpha\in A\cap B.\end{cases}\end{aligned}$$

We consider the case, when $p^{-1}(\beta)\cap(A\cup B)\neq\phi$, as otherwise it is trivial. Then

$$f\left((F,A)\,\widetilde{\cup}\,(G,B)\right)\beta=u\left(\bigcup\begin{cases} F(\alpha), & \alpha\in(A-B)\cap p^{-1}(\beta)\\ G(\alpha), & \alpha\in(B-A)\cap p^{-1}(\beta)\\ F(\alpha)\cup G(\alpha), & \alpha\in(A\cap B)\cap p^{-1}(\beta)\end{cases}\right) \qquad \text{(I)}$$

Next, for the non-trivial case, using Definition 11 and for $\beta \in E'$, we have

$$\begin{aligned}
\left(f(F,A)\ \widetilde{\cup}\ f(G,B)\right)\beta &= f(F,A)\beta\ \cup\ f(G,B)\beta \\
&= u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha)\right)\cup u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap B} G(\alpha)\right) \\
&= u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha)\cup \bigcup_{\alpha\in p^{-1}(\beta)\cap B} G(\alpha)\right) \\
&= u\left(\bigcup\begin{cases} F(\alpha), & \alpha\in(A-B)\cap p^{-1}(\beta) \\ G(\alpha), & \alpha\in(B-A)\cap p^{-1}(\beta) \\ F(\alpha)\cup G(\alpha), & \alpha\in(A\cap B)\cap p^{-1}(\beta). \end{cases}\right) \quad \text{(II)}
\end{aligned}$$

From (I) and (1), we have (3).

(4) For $\beta \in E'$, we show that $f\left((F,A)\,\widetilde{\cap}\,(G,B)\right)\beta \subseteq \left(f(F,A)\ \widetilde{\cap}\ f(G,B)\right)\beta$. Consider

$$f\left((F,A)\,\widetilde{\cap}\,(G,B)\right)\beta = f(H,A\cap B)\beta = \begin{cases} u\left(\displaystyle\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} H(\alpha)\right), & \text{if } p^{-1}(\beta)\cap(A\cap B)\neq\phi, \\ \phi, & \text{otherwise,}\end{cases}$$

where $H(\alpha) = F(\alpha)\cap G(\alpha)$. We consider the case when $p^{-1}(\beta)\cap(A\cap B)\neq\phi$, as otherwise it is trivial. Thus

$$f(H,A\cap B)\beta = u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} H(\alpha)\right) = u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} (F(\alpha)\cap G(\alpha))\right),$$

or

$$f\left((F,A)\,\widetilde{\cap}\,(G,B)\right)\beta = u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} (F(\alpha)\cap G(\alpha))\right) \quad \text{(I)}$$

On the other hand, using Definition 11, we have

$$\begin{aligned}
\left(f((F,A))\ \widetilde{\cap}\ f((G,B))\right)\beta &= f(F,A)\beta\ \cap\ f(G,B)\beta \\
&= \left(\begin{cases} u\left(\displaystyle\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha)\right), & \text{if } p^{-1}(\beta)\cap A\neq\phi, \\ \phi, & \text{otherwise.}\end{cases}\right)\cap \\
&\quad \left(\begin{cases} u\left(\displaystyle\bigcup_{\alpha\in p^{-1}(\beta)\cap B} G(\alpha)\right), & \text{if } p^{-1}(\beta)\cap B\neq\phi, \\ \phi, & \text{otherwise.}\end{cases}\right)
\end{aligned}$$

Ignoring the trivial case, we get

$$\begin{aligned}\left(f\left(F,A\right)\widetilde{\cap}f\left(G,B\right)\right)\beta &= u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap A}F\left(\alpha\right)\right)\ \cap\ u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap B}G\left(\alpha\right)\right)\\ &\supseteq u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)}\left(F\left(\alpha\right)\cap G\left(\alpha\right)\right)\right)=f\left(\left(F,A\right)\ \widetilde{\cap}\ \left(G,B\right)\right)\beta\end{aligned}$$

(5) For $\beta\in E'$

$$f\left(F,A\right)\beta=\begin{cases}u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap A}F\left(\alpha\right)\right), & \text{if } p^{-1}\left(\beta\right)\cap A\neq\phi,\\ \phi, & \text{otherwise.}\end{cases}$$

We consider the case when $p^{-1}\left(\beta\right)\cap A\neq\phi$, as otherwise it is trivial. Then

$$f\left(F,A\right)\beta=u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap A}F\left(\alpha\right)\right)\subseteq u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap B}G\left(\alpha\right)\right)=f\left(G,B\right)\beta.$$

This gives (5).

■

In Theorem 14, inequalities (2) and (4), cannot be reversed, in general, as is shown in the following:

Example 15. *The soft classes $(X,E),(Y,E)$ and mapping $f:(X,E)\rightarrow(Y,E')$ are as defined in Example 10. Then*

$$\widetilde{Y}\not\subseteq f\left(\widetilde{X}\right)=\left\{e_1'=\{x,z\},e_2'=\{x,z\},e_3'=\{x,z\}\right\}.$$

This shows that the reversal of inequality (2) is not true.
To show that the reversal of (4) does not hold, choose soft sets in (X,E) as:

$$\begin{aligned}(F,A) &= \{e_1=\{c\},e_2=\{b,c\},e_3=\{a,b,c\}\},\\ (G,B) &= \{e_1=\{a\},e_2=\{a,c\},e_3=\{b\},e_4=\{b,c\}\}.\end{aligned}$$

Then calculations show that

$$f\left(F,A\right)\widetilde{\cap}f\left(G,B\right)=\{\{e_2'=\{z\}\},e_3'=\{y,z\}\}\,\widetilde{\not\subseteq}\,\{\{e_2'=\{z\},e_3'=\{y\}\}\}=f\left(\left(F,A\right)\widetilde{\cap}\left(G,B\right)\right).$$

Theorem 16. *Let $f:(X,E)\rightarrow(Y,E')$, $u:X\rightarrow Y$ and $p:E\rightarrow E'$ be mappings. Then for soft sets (F,A), (G,B) and a family of soft sets (F_i,A_i) in the soft class (Y,E'), we have:*
(1) $f^{-1}\left(\widetilde{\Phi}\right)=\widetilde{\Phi}$.
(2) $f^{-1}\left(\widetilde{Y}\right)=\widetilde{X}$.

(3) $f^{-1}\left((F,A)\ \widetilde{\cup}\ (G,B)\right) = f^{-1}(F,A)\ \widetilde{\cup}\ f^{-1}(G,B)$.
In general $f^{-1}\left(\widetilde{\bigcup_i}(F_i,A_i)\right) = \widetilde{\bigcup_i} f^{-1}(F_i,A_i)$.
(4) $f^{-1}\left((F,A)\ \widetilde{\cap}\ (G,B)\right) = f^{-1}(F,A)\ \widetilde{\cap}\ f^{-1}(G,B)$.
In general $f^{-1}\left(\widetilde{\bigcap_i}(F_i,A_i)\right) = \widetilde{\bigcap_i} f^{-1}(F_i,A_i)$.
(5) *If* $(F,A)\,\widetilde{\subseteq}\,(G,B)$, *then* $f^{-1}(F,A)\ \widetilde{\subseteq}\ f^{-1}(G,B)$.

Proof. We only prove $(3)-(5)$.
(3) For $\alpha \in E$

$$\begin{aligned} f^{-1}\left((F,A)\,\widetilde{\cup}\,(G,B)\right)\alpha &= f^{-1}(H, A\cup B)\,\alpha \\ &= u^{-1}(H(p(\alpha))),\ p(\alpha)\in A\cup B \\ &= u^{-1}(H(\beta)),\ \text{where } \beta = p(\alpha) \\ &= u^{-1}\left(\begin{cases} F(\beta), & \beta\in A-B \\ G(\beta), & \beta\in B-A \\ F(\beta)\cup G(\beta), & \beta\in A\cap B \end{cases}\right) \qquad \text{(I)} \end{aligned}$$

Next, using Definition 12, we have

$$\begin{aligned} \left(f^{-1}(F,A)\ \widetilde{\cup}\ f^{-1}(G,B)\right)\alpha &= f^{-1}(F,A)\,\alpha \cup f^{-1}(G,B)\,\alpha \\ &= u^{-1}(F(p(\alpha)))\ \cup\ u^{-1}(G(p(\alpha))),\ p(\alpha)\in A\cap B \\ &= u^{-1}\left(\begin{cases} F(\beta), & \beta\in A-B \\ G(\beta), & \beta\in B-A \\ F(\beta)\cup G(\beta), & \beta\in A\cap B \end{cases}\right) \qquad \text{(II)} \end{aligned}$$

where $\beta = p(\alpha)$.
From (1) and (1), we obtain (3).

(4) For $\alpha \in E$

$$\begin{aligned} \left(f^{-1}\left((F,A)\ \widetilde{\cap}\ (G,B)\right)\right)\alpha &= \left(f^{-1}(H, A\cap B)\right)\alpha \\ &= u^{-1}(H(p(\alpha))),\ p(\alpha)\in A\cap B \\ &= u^{-1}(F(\beta)\cap G(\beta)),\ \text{where } \beta = p(\alpha). \\ &= u^{-1}(F(\beta))\cap u^{-1}(G(\beta)) \\ &= u^{-1}(F(p(\alpha)))\cap u^{-1}(G(p(\alpha))) \\ &= \left(f^{-1}(F,A)\,\widetilde{\cap} f^{-1}(G,B)\right)\alpha. \end{aligned}$$

This proves (4).

(5) For $\alpha \in E$, consider

$$\begin{aligned} f^{-1}(F,A)\,\alpha &= u^{-1}(F(p(\alpha))) = u^{-1}(F(\beta)),\ \text{where } \beta = p(\alpha) \\ &\subseteq u^{-1}(G(\beta)) = u^{-1}(G(p(\alpha))) = f^{-1}(G,B)\,\alpha. \end{aligned}$$

This gives (5). ■

4. An Application in Medical Expert Systems

An important task of a medical expert system is to transform a patient's complaints/symptoms into a set of possible causes and their respective importance from the view point of a medical specialist. A patient's case may easily be encoded into a soft set. Suppose following is the narration by the patient:

> I have three main complaints viz. burning in stomach, headache and sleeplessness. Whatever sleep I get, is unrefreshing and semi-conscious. I mean, I am always aware what is transpiring in the room when asleep. I have some pain in joints and backbone. To a less degree I also suffer from depression and anxiety.

This can be written as the following soft set:

$$(F,A)=\left\{\begin{array}{c}\text{high importance}=\{\text{burning in stomach, headache, sleeplessness}\}\,,\\ \text{medium importance}=\{\text{semi-coscious sleep}\}\,,\\ \text{low importance}=\{\text{joint pain, backbone pain, depression, anxiety}\}\end{array}\right\}$$

The medical knowledge may be encoded in the form of a look-up tables. Look-up tables are the computer representation of the notion of mapping in mathematics. Suppose our medical experts have provided us with following knowledge:

$$\begin{array}{lcllcl} u\,(\text{burning in stomach}) & = & \text{acidity}, & u\,(\text{headache}) & = & \text{blood pressure},\\ u\,(\text{sleeplessness}) & = & \text{acidity}, & u\,(\text{backbone pain}) & = & \text{wrong posture},\\ u\,(\text{semi-conscious sleep}) & = & \text{fatigue}, & u\,(\text{joint pain}) & = & \text{acidity},\\ u\,(\text{depression}) & = & \text{low energy level}, & u\,(\text{anxiety}) & = & \text{low energy level}, \end{array}$$

and

$$p\,(\text{high importance})=\text{infrequent high potency},\quad p\,(\text{medium importance})=\text{frequent low potency}\ .$$

For the sake of ease in mathematical manipulation we denote the symptoms and gradations by symbols as follows:

$$\begin{array}{lcl} b & = & \text{burning in stomach}\\ h & = & \text{headache}\\ s & = & \text{sleeplessness}\\ c & = & \text{semi-conscious sleep}\\ j & = & \text{joint pain}\\ p & = & \text{backbone pain}\\ d & = & \text{depression}\\ a & = & \text{anxiety} \end{array}\ ,\quad \begin{array}{lcl} e_1 & = & \text{high importance}\\ e_2 & = & \text{medium importance}\\ e_3 & = & \text{low importance} \end{array}$$

and

$$\begin{array}{lcl} \alpha & = & \text{acidity}\\ \beta & = & \text{blood pressure}\\ \gamma & = & \text{fatigue}\\ \delta & = & \text{wrong posture}\\ \lambda & = & \text{depression}\\ \mu & = & \text{mood disorder} \end{array}\ ,\quad \begin{array}{lcl} e_1' & = & \text{infrequent high potency}\\ e_2' & = & \text{frequent low potency} \end{array}\ .$$

Thus we have two soft classes (X,E) and (Y,E') with $X=\{b,h,s,c,j,p,d,a\}$, $E=\{e_1,e_2,e_3\}$ and $Y=\{\alpha,\beta,\gamma,\delta,\lambda,\mu\}$, $E'=\{e_1',e_2'\}$. (X,E) is the soft class of symptoms and their importance for the patient, and (Y,E') represents causes and medical preference for treatment. The soft set of patient's narration may be given as:

$$(F,A)=\{e_1=\{b,h,s\},e_2=\{c\},e_3=\{j,p,d,a\}\}.$$

As a first task of the medical expert system, stored medical knowledge is to be applied on the given case. This knowledge, in the language of computer programming, is given as look-up tables. Mappings $u:X\to Y$ and $p:E\to E'$ are defined as:

$$\begin{aligned} u(b) &= \alpha,\ u(h)=\beta,\ u(s)=\alpha,\ u(c)=\gamma,\ u(j)=\alpha,\ u(p)=\delta,\ u(d)=\mu,\ u(a)=\mu, \\ p(e_1) &= e_1',\ p(e_2)=e_2'. \end{aligned}$$

Calculations give:

$$\begin{aligned} f(F,A) &= \{e_1'=\{\alpha,\beta\},e_2'=\{\gamma\}\} \\ &= \{\text{infrequent high potency}=\{\text{acidity, blood pressure}\},\ \text{frequent low potency}\ =\{\text{fatigue}\}\}. \end{aligned}$$

Conclusion 17. *A soft set, being a collection of information granules, is the mathematical formulation of approximate reasoning about information systems. In this paper, we define the notion of mapping on soft classes. Several properties of soft images and soft inverse images have been established and supported by examples and counterexamples. Finally, these notions have been applied to the problem of medical diagnosis. It is hoped that these notions will be useful for the researchers to further promote and advance this research in Soft Set Theory.*

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